Tutorial 10

Exercises 16.5

19. Let $x = r\cos\theta$, $y = r\sin\theta$, then $z = 2r \in [2,6] \Rightarrow r \in [1,3]$, and $\theta \in [0, 2\pi]$. Thus,
$\vec{r}(r,\theta) = r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j} + 2r\,\vec{k}$

$\Rightarrow \vec{r}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j} + 2\vec{k}$, $\vec{r}_\theta = -r\sin\theta\,\vec{i} + r\cos\theta\,\vec{j}$

$\Rightarrow$
$$\vec{r}_r \times \vec{r}_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 2 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = (-2r\cos\theta)\vec{i} + (-2r\sin\theta)\vec{j} + r\vec{k}$$

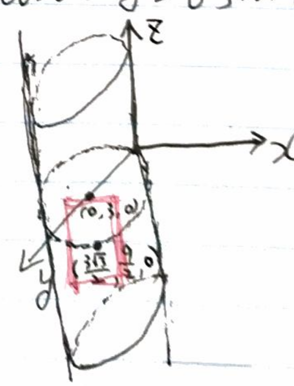
$\Rightarrow |\vec{r}_r \times \vec{r}_\theta| = \sqrt{5}\,r$

$\Rightarrow$ Area $= \int_0^{2\pi}\int_1^3 \sqrt{5}\,r\,dr\,d\theta = \sqrt{5}\int_0^{2\pi} \frac{1}{2}r^2\Big|_1^3 d\theta = 4\sqrt{5}\int_0^{2\pi} d\theta = 8\sqrt{5}\pi$

29. Let $\begin{cases} 3\sin 2\theta = 3\cdot\frac{\sqrt{3}}{2} \\ 6\sin^2\theta = \frac{9}{2} \\ z = 0 \end{cases} \Rightarrow \begin{cases} \theta = \frac{\pi}{3} \\ z = 0 \end{cases} \Rightarrow$ At P_0, $\vec{r}_\theta = (6\cos 2\theta)\vec{i} + 12\sin\theta\cos\theta\,\vec{j} = -3\vec{i} + 3\sqrt{3}\,\vec{j}$
$\vec{r}_z = \vec{k}$

$$\Rightarrow \vec{r}_\theta \times \vec{r}_z = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 3\sqrt{3} & 0 \\ 0 & 0 & 1 \end{vmatrix} = 3\sqrt{3}\,\vec{i} + 3\vec{j} \Rightarrow \text{the tangent plane is}$$
$$(3\sqrt{3}\,\vec{i} + 3\vec{j})\cdot\left((x - \tfrac{3\sqrt{3}}{2})\vec{i} + (y - \tfrac{9}{2})\vec{j} + z\vec{k}\right) = 0$$

$\Rightarrow \sqrt{3}x + y = 9$. The parametrization $\Rightarrow x = 3\sin 2\theta = 6\sin\theta\cos\theta$, $y = 6\sin^2\theta$
$\Rightarrow x^2 + y^2 = 36\sin^2\theta = 6y \Rightarrow x^2 + (y-3)^2 = 9$



41. $\vec{p} = \vec{k}$, $F(x,y,z) = x^2 - 2y - 2z = 0$,
$\Rightarrow \nabla F = (2x, -2, -2) \Rightarrow \nabla F \cdot \vec{p} = -2$
$|\nabla F| = \sqrt{4x^2 + 8} = 2\sqrt{x^2+2}$

$\Rightarrow$
$$S = \iint_R \frac{|\nabla F|}{|\nabla F \cdot \vec{p}|}\,dA = \iint_R \frac{2\sqrt{x^2+2}}{2}\,dx\,dy$$
$$= \int_0^2\int_0^{3x} \sqrt{x^2+2}\,dy\,dx = 3\int_0^2 x\sqrt{x^2+2}\,dx$$
$$= \frac{3}{2}\int_0^2 \sqrt{x^2+2}\,d(x^2+2) = (x^2+2)^{\frac{3}{2}}\Big|_0^2 = 6^{\frac{3}{2}} - 2^{\frac{3}{2}}$$